GLOBAL EDITION

Essentials of College Algebra

ELEVENTH EDITION

Margaret L. Lial • John Hornsby • David I. Schneider • Callie J. Daniels

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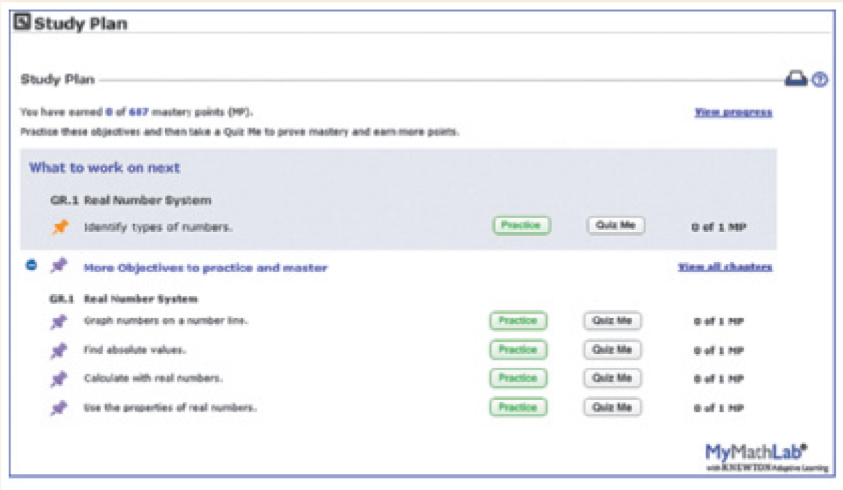

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Essentials of College Algebra

GLOBAL EDITION

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To Faith Elizabeth Varnado—you are the light of our lives
E.J.H.

To Kurtis, Clayton, and Grady
C.J.D.

To Margaret L. Lial

On March 16, 2012, the mathematics education community lost one of its most influential members with the passing of our beloved mentor, colleague, and friend Marge Lial. On that day, Marge lost her long battle with ALS. Throughout her illness, Marge showed the remarkable strength and courage that characterized her entire life.

We would like to share a few comments from among the many messages we received from friends, colleagues, and others whose lives were touched by our beloved Marge:

"What a lady"
"A remarkable person"
"Gracious to everyone"
"One of a kind"
"Truly someone special"
"A loss in the mathematical world"
"A great friend"
"Sorely missed but so fondly remembered"
"Even though our crossed path was narrow, she made an impact and I will never forget her."
"There is talent and there is Greatness. Marge was truly Great."
"Her true impact is almost more than we can imagine."

In the world of college mathematics publishing, Marge Lial was a rock star. People flocked to her, and she had a way of making everyone feel like they truly mattered. And to Marge, they did. She and Chuck Miller began writing for Scott Foresman in 1970. Three years before her passing she told us that she could no longer continue because "just getting from point A to point B" had become too challenging. That's our Marge—she even gave a geometric interpretation to her illness.

It has truly been an honor and a privilege to work with Marge Lial these past twenty years. While we no longer have her wit, charm, and loving presence to guide us, so much of who we are as mathematics educators has been shaped by her influence. We will continue doing our part to make sure that the Lial name represents excellence in mathematics education. And we remember daily so many of the little ways she impacted us, including her special expressions, "Margisms" as we like to call them. She often ended emails with one of them—the single word "Onward."

We conclude with a poem by Callie Daniels describing Marge's influence on her and many others in the mathematics community.

Your courage inspires me
Your strength...impressive
Your wit humors me
Your vision...progressive

Your determination motivates me
Your accomplishments pave my way
Your vision sketches images for me
Your influence will forever stay.

Thank you, dearest Marge.
Knowing you and working with you has been a divine gift.

Onward.

John Hornsby
David Schneider
Callie Daniels

Contents

Preface

WELCOME TO THE 11TH EDITION

As authors, we have called upon our classroom experiences, use of MyMathLab, suggestions from users and reviewers, and many years of writing to provide tools that will support learning and teaching. This new edition of *Essentials of College Algebra* continues our effort to provide a sound pedagogical approach through logical development of the subject matter. This approach forms the basis for all of the Lial team's instructional materials available from Pearson Education, in both print and technology forms.

Our goal is to produce a textbook that will be an integral component of the student's experience in learning college algebra. With this in mind, we have provided a textbook that students can read more easily, which is often a difficult task, given the nature of mathematical language. We have also improved page layouts for better flow, provided additional side comments, and updated many figures.

We realize that today's classroom experience is evolving and that technology-based teaching and learning aids have become essential to address the ever-changing needs of instructors and students. As a result, we've worked to provide support for all classroom types—traditional, hybrid, and online. In the 11th edition, text and online materials are more tightly integrated than ever before. This enhances flexibility and ease of use for instructors and increases success for students. See pages 14–15 for descriptions of these materials.

NEW TO THE 11TH EDITION

- Enhancing the already well-respected exercises, approximately 750 are new or modified, and hundreds present updated real-life data. In addition, the MyMathLab course has expanded coverage of all exercise types appearing in the exercise sets, as well as the mid-chapter Quizzes and Summary Exercises.
- In **Chapter 1** we have rewritten the introduction to the set of complex numbers and have included a new diagram illustrating the relationships among its subsets. We have also expanded the discussion of solving absolute value equations and inequalities.
- In **Chapter 2** we have prepared new art for shrinking and stretching graphs, added new exercises on translations, and expanded the exercises involving the difference quotient. We now use open intervals when discussing increasing, decreasing, and constant behavior of functions, and we now define intercepts as ordered pairs.
- **Chapter 3** has undergone a particularly extensive revision. We have updated the opening discussion by providing a table of examples of polynomial functions (constant, linear, quadratic, cubic, quartic) and have identified the degree and leading coefficient. There is an increased emphasis on displaying all possibilities for positive, negative, and nonreal complex zeros in a table format, with graphical representations to illustrate them. We now have a new summary exercise set on solving both equations and inequalities (linear, quadratic, polynomial, rational, and miscellaneous).
- In **Chapter 4** we have added more work with translations of exponential and logarithmic functions, and we have included additional examples and exercises on solving equations that involve these functions.

- In **Chapter 5** we now present the determinant theorems within the exposition, and we have written new examples and exercises showing how they are used.
- For visual learners, numbered **Figure** and **Example** references within the text are set using the same typeface as the figure and bold print for the example. This makes it easier for the students to identify and connect them. We also have increased our use of a "drop down" style, when appropriate, to distinguish between simplifying expressions and solving equations, and we have added many more explanatory side comments. Interactive figures with accompanying exercises and explorations are now available and assignable in MyMathLab.

FEATURES OF THIS TEXT

SUPPORT FOR LEARNING CONCEPTS

We provide a variety of features to support students' learning of the essential topics of college algebra. Explanations that are written in understandable terms, figures and graphs that illustrate examples and concepts, graphing technology that supports and enhances algebraic manipulations, and real-life applications that enrich the topics with meaning all provide opportunities for students to deepen their understanding of mathematics. These features help students make mathematical connections and expand their own knowledge base.

- **Examples** Numbered examples that illustrate the techniques for working exercises are found in every section. We use traditional explanations, side comments, and pointers to describe the steps taken—and to warn students about common pitfalls. Some examples provide additional graphing calculator solutions, although these can be omitted if desired.
- **Now Try Exercises** Following each numbered example, the student is directed to try a corresponding odd-numbered exercise (or exercises). This feature allows for quick feedback to determine whether the student has understood the principles illustrated in the example.
- **Real-Life Applications** We have included hundreds of real-life applications, many with data updated from the previous edition. They come from fields such as business, entertainment, sports, biology, astronomy, geology, and environmental studies.
- **Function Boxes** Beginning in Chapter 2, functions provide a unifying theme throughout the text. Special function boxes (for example, see page 248) offer a comprehensive, visual introduction to each type of function and also serve as an excellent resource for reference and review. Each function box includes a table of values, traditional and calculator-generated graphs, the domain, the range, and other special information about the function. These boxes are now assignable in MyMathLab.
- **Figures and Photos** Today's students are more visually oriented than ever before, and we have updated the figures in this edition to a greater extent than in our previous few editions. Interactive figures with accompanying exercises and explorations are now available and assignable in MyMathLab.
- **Use of Graphing Technology** We have integrated the use of graphing calculators where appropriate, although ***this technology is completely optional and can be omitted without loss of continuity***. We continue to stress that graphing calculators support understanding but that students must first master the underlying mathematical concepts. Exercises that require their use are marked with an icon .

- **Cautions and Notes** Text that is marked **CAUTION** warns students of common errors, and **NOTE** comments point out explanations that should receive particular attention.
- **Looking Ahead to Calculus** These margin notes offer glimpses of how the topics currently being studied are used in calculus.

SUPPORT FOR PRACTICING CONCEPTS

This text offers a wide variety of exercises to help students master college algebra. The extensive exercise sets provide ample opportunity for practice, and the exercise problems increase in difficulty so that students at every level of understanding are challenged. The variety of exercise types promotes understanding of the concepts and reduces the need for rote memorization.

- **Exercise Sets** We have revised many drill and application exercises for better pairing of corresponding even and odd exercises, and answers to the odd exercises are provided in this edition. In addition to these, we include writing exercises, optional graphing calculator problems, and multiple-choice, matching, true/false, and completion exercises. Those marked ***Concept Check*** focus on conceptual thinking. ***Connecting Graphs with Equations*** exercises challenge students to write equations that correspond to given graphs. Finally, MyMathLab offers Pencast solutions for selected Connecting Graphs with Equations problems.
- **Relating Concepts Exercises** Appearing in selected exercise sets, these groups of exercises are designed so that students who work them in numerical order will follow a line of reasoning that leads to an understanding of how various topics and concepts are related. All answers to these exercises appear in the student answer section, and these exercises are now assignable in MyMathLab.
- **Complete Solutions to Selected Exercises** Exercise numbers marked indicate that a full worked-out solution appears at the back of the text. These are often exercises that extend the skills and concepts presented in the numbered examples.

SUPPORT FOR REVIEW AND TEST PREP

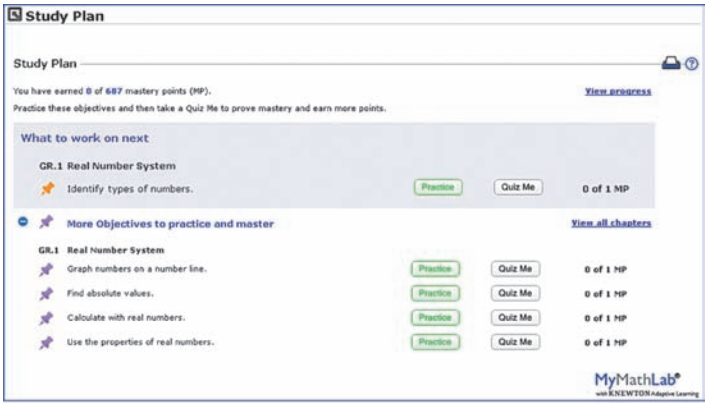
Ample opportunities for review are found within the chapters and at the ends of chapters. Quizzes that are interspersed within chapters provide a quick assessment of students' understanding of the material presented up to that point in the chapter. Chapter "Test Preps" provide comprehensive study aids to help students prepare for tests.

- **Quizzes** Students can periodically check their progress with in-chapter quizzes that appear in all chapters, beginning with Chapter 1. All answers, with corresponding section references, appear in the student answer section. These quizzes and additional cumulative chapter review exercises are now assignable in MyMathLab.
- **Summary Exercises** These sets of in-chapter exercises give students the all-important opportunity to work *mixed* review exercises, requiring them to synthesize concepts and select appropriate solution methods. The summary exercises are now assignable in MyMathLab.
- **End-of-Chapter Test Prep** Following the final numbered section in each chapter, the Test Prep provides a list of **Key Terms,** a list of **New Symbols** (if applicable), and a two-column **Quick Review** that includes a section-by-section summary of concepts and examples. This feature concludes with a comprehensive set of **Review Exercises** and a **Chapter Test.** The Test Prep, Review Exercises, and Chapter Test are assignable in MyMathLab.
- **Glossary** A comprehensive glossary of important terms drawn from the entire book follows Chapter 5.

Resources for Success

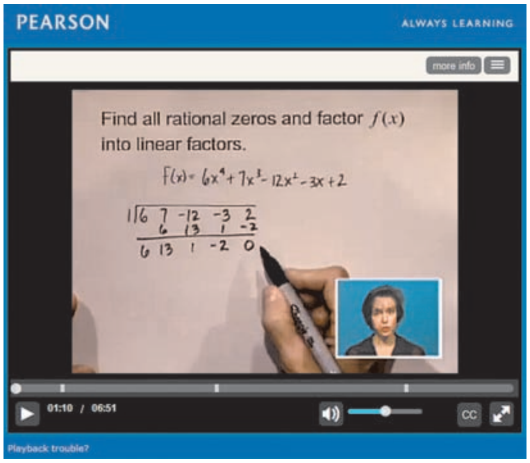
MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from an **experienced partner** with educational expertise and an eye on the future. MyMathLab helps prepare students and gets them thinking more conceptually and visually through the following features:



Adaptive Study Plan

The Study Plan makes studying more efficient and effective for every student. Performance and activity are assessed continually in real time. The data and analytics are used to provide personalized, content–reinforcing concepts that target each student's strengths and weaknesses.

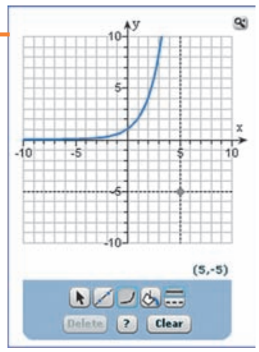


Video Assessment

Video assessment is tied to key Example/Solution videos to check students' conceptual understanding of important math concepts.

Enhanced Graphing Functionality

New functionality within the graphing utility allows graphing of 3-point quadratic functions, 4-point cubic graphs, and transformations in exercises.



Skills for Success Modules are integrated within the MyMathLab course to help students succeed in collegiate courses and prepare for future professions.

Cumulative Review Assignments in MyMathLab help students synthesize and maintain concepts.

Instructor Resources

Additional resources can be downloaded from **www.pearsonglobaleditions.com/Lial.**

TestGen®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides

These fully editable slides correlate to the textbook.

Instructor's Solutions Manual

This manual includes fully worked solutions to all textbook exercises.

Instructor's Testing Manual

This manual includes diagnostic pretests, chapter tests, final exams, and additional test items, with answers provided.

Student Resources

The following are additional resources to help improve student success.

Video Lectures

Quick Review videos cover key definitions and procedures from each section. Example/Solution videos walk students through the detailed solution process for every example in the textbook.

Additional Skill and Drill Manual

This manual provides additional practice and test preparation for students.

MyNotes

This note-taking guide is available in MyMathLab or packaged with text and access code. It offers structure for student reading and understanding of the textbook. It includes textbook examples along with ample space for students to write solutions and notes.

MyClassroomExamples

This alternate version of a note-taking guide is available in MyMathLab and offers structure for classroom lecture. It includes Classroom Examples along with ample space for students to write solutions and notes.

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As an author team, we are committed to providing the best possible college algebra course to help instructors teach and students succeed. As we continue to work toward this goal, we welcome any comments or suggestions you might send, via e-mail, to math@pearson.com.

John Hornsby
David I. Schneider
Callie J. Daniels

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R Review of Basic Concepts

Positive and negative numbers, used to represent gains and losses on a board such as this one, are examples of *real numbers* encountered in applications of mathematics.

R.1 Sets

- Basic Definitions
- Operations on Sets

Basic Definitions

A **set** is a collection of objects. The objects that belong to a set are called the **elements,** or **members,** of the set. In algebra, the elements of a set are usually numbers. Sets are commonly written using **set braces,** $\{\ \}$. For example, the set containing the elements 1, 2, 3, and 4 is written as follows.

$$\{1, 2, 3, 4\}$$

Since the order in which the elements are listed is not important, this same set can also be written as $\{4, 3, 2, 1\}$ or with any other arrangement of the four numbers.

To show that 4 is an element of the set $\{1, 2, 3, 4\}$, we use the symbol $\in$.

$$4 \in \{1, 2, 3, 4\}$$

Since 5 is *not* an element of this set, we place a slash through the symbol $\in$.

$$5 \notin \{1, 2, 3, 4\}$$

It is customary to name sets with capital letters. If S is used to name the set above, then we write it as follows.

$$S = \{1, 2, 3, 4\}$$

Set S was written by listing its elements. Set S might also be described as

"the set containing the first four counting numbers."

In this example, the notation $\{1, 2, 3, 4\}$, with the elements listed between set braces, is briefer than the verbal description.

The set F, consisting of all fractions between 0 and 1, is an example of an **infinite set,** one that has an unending list of distinct elements. A **finite set** is one that has a limited number of elements. The process of counting its elements comes to an end. Some infinite sets can be described by listing. For example, the set of numbers N used for counting, called the **natural numbers,** or the **counting numbers,** can be written as follows.

$$\boldsymbol{N = \{1, 2, 3, 4, \ldots\}} \quad \text{Natural (counting) numbers}$$

The three dots (*ellipsis points*) show that the list of elements of the set continues according to the established pattern.

Sets are often written using a variable to represent an arbitrary element of the set. For example,

$$\{x \mid x \text{ is a natural number between 2 and 7}\} \quad \text{Set-builder notation}$$

(which is read "the set of all elements x such that x is a natural number between 2 and 7") uses **set-builder notation** to represent the set $\{3, 4, 5, 6\}$. The numbers 2 and 7 are *not* between 2 and 7.

EXAMPLE 1 Using Set Notation and Terminology

Identify each set as *finite* or *infinite*. Then determine whether 10 is an element of the set.

(a) $\{7, 8, 9, \ldots, 14\}$

(b) $\{1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots\}$

(c) $\{x \mid x \text{ is a fraction between 1 and 2}\}$

(d) $\{x \mid x \text{ is a natural number between 9 and 11}\}$

SOLUTION

(a) The set is finite, because the process of counting its elements 7, 8, 9, 10, 11, 12, 13, and 14 comes to an end. The number 10 does belong to the set, and this is written as follows.

$$10 \in \{7, 8, 9, \ldots, 14\}$$

(b) The set is infinite, because the ellipsis points indicate that the pattern continues forever. In this case,

$$10 \notin \left\{1, \tfrac{1}{4}, \tfrac{1}{16}, \tfrac{1}{64}, \ldots\right\}.$$

(c) Between any two distinct natural numbers there are infinitely many fractions, so this set is infinite. The number 10 is not an element.

(d) There is only one natural number between 9 and 11, namely 10. So the set is finite, and 10 is an element.

✔ *Now Try Exercises 1, 3, 5, and 7.*

EXAMPLE 2 Listing the Elements of a Set

Use set notation, and write the elements belonging to each set.

(a) $\{x \mid x \text{ is a natural number less than } 5\}$

(b) $\{x \mid x \text{ is a natural number greater than 7 and less than } 14\}$

SOLUTION

(a) The natural numbers less than 5 form the set $\{1, 2, 3, 4\}$.

(b) This is the set $\{8, 9, 10, 11, 12, 13\}$.

✔ *Now Try Exercise 15.*

When we are discussing a particular situation or problem, the **universal set** (whether expressed or implied) contains all the elements included in the discussion. The letter U is used to represent the universal set. The **null set,** or **empty set,** is the set containing no elements. We write the null set by either using the special symbol $\emptyset$, or else writing set braces enclosing no elements, $\{\ \}$.

CAUTION Do not combine these symbols. $\{\emptyset\}$ ***is not the null set.***

Every element of the set $S = \{1, 2, 3, 4\}$ is a natural number. S is an example of a *subset* of the set N of natural numbers, and this is written

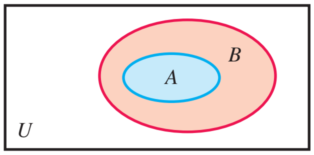
$$S \subseteq N.$$

By definition, set A is a **subset** of set B if every element of set A is also an element of set B. For example, if $A = \{2, 5, 9\}$ and $B = \{2, 3, 5, 6, 9, 10\}$, then $A \subseteq B$. However, there are some elements of B that are not in A, so B is not a subset of A, which is written

$$B \nsubseteq A.$$

By the definition, every set is a subset of itself. Also, by definition, $\emptyset$ is a subset of every set.

If A is any set, then $\emptyset \subseteq A$.



$A \subseteq B$

Figure 1

Figure 1 shows a set A that is a subset of set B. The rectangle in the drawing represents the universal set U. Such diagrams are called **Venn diagrams.**

Two sets A and B are equal whenever $A \subseteq B$ and $B \subseteq A$. Equivalently, $A = B$ if the two sets contain exactly the same elements. For example,

$$\{1, 2, 3\} = \{3, 1, 2\}$$

is true, since both sets contain exactly the same elements. However,

$$\{1, 2, 3\} \neq \{0, 1, 2, 3\},$$

since the set $\{0, 1, 2, 3\}$ contains the element 0, which is not an element of $\{1, 2, 3\}$.

EXAMPLE 3 Examining Subset Relationships

Let $U = \{1, 3, 5, 7, 9, 11, 13\}$, $A = \{1, 3, 5, 7, 9, 11\}$, $B = \{1, 3, 7, 9\}$, $C = \{3, 9, 11\}$, and $D = \{1, 9\}$. Determine whether each statement is *true* or *false*.

(a) $D \subseteq B$ **(b)** $B \subseteq D$ **(c)** $C \nsubseteq A$ **(d)** $U = A$

SOLUTION

(a) All elements of D, namely 1 and 9, are also elements of B, so D is a subset of B, and $D \subseteq B$ is true.

(b) There is at least one element of B (for example, 3) that is not an element of D, so B is *not* a subset of D. Thus, $B \subseteq D$ is false.

(c) C is a subset of A, because every element of C is also an element of A. Thus, $C \subseteq A$ is true, and as a result, $C \nsubseteq A$ is false.

(d) U contains the element 13, but A does not. Therefore, $U = A$ is false.

✔ *Now Try Exercises 43, 45, 53, and 55.*

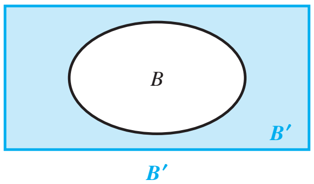


Figure 2

Operations on Sets Given a set A and a universal set U, the set of all elements of U that do not belong to set A is called the **complement** of set A. For example, if set A is the set of all students in your class 30 years old or older, and set U is the set of all students in the class, then the complement of A would be the set of all the students in the class younger than age 30. The complement of set A is written A' (read **"A-prime"**). The Venn diagram in **Figure 2** shows a set B. Its complement, B', is in color.

EXAMPLE 4 Finding the Complement of a Set

Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5, 7\}$, and $B = \{3, 4, 6\}$. Find each set.

(a) A' **(b)** B' **(c)** $\emptyset'$ **(d)** U'

SOLUTION

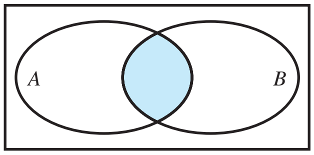
(a) Set A' contains the elements of U that are not in A. Thus, $A' = \{2, 4, 6\}$.

(b) $B' = \{1, 2, 5, 7\}$ **(c)** $\emptyset' = U$ **(d)** $U' = \emptyset$

✔ *Now Try Exercise 79.*

Given two sets A and B, the set of all elements belonging both to set A ***and*** to set B is called the **intersection** of the two sets, written $A \cap B$. For example, if $A = \{1, 2, 4, 5, 7\}$ and $B = \{2, 4, 5, 7, 9, 11\}$, then we have the following.

$$A \cap B = \{1, 2, 4, 5, 7\} \cap \{2, 4, 5, 7, 9, 11\} = \{2, 4, 5, 7\}$$



$A \cap B$

Figure 3

The Venn diagram in **Figure 3** shows two sets A and B. Their intersection, $A \cap B$, is in color.

Two sets that have no elements in common are called **disjoint sets.** If A and B are any two disjoint sets, then $A \cap B = \emptyset$. For example, there are no elements common to both $\{50, 51, 54\}$ and $\{52, 53, 55, 56\}$, so these two sets are disjoint.

$$\{50, 51, 54\} \cap \{52, 53, 55, 56\} = \emptyset$$

EXAMPLE 5 **Finding the Intersection of Two Sets**

Find each of the following.

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\}$

(b) $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\}$

(c) $\{1, 3, 5\} \cap \{2, 4, 6\}$

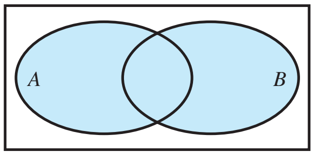
SOLUTION

(a) $\{9, 15, 25, 36\} \cap \{15, 20, 25, 30, 35\} = \{15, 25\}$
The elements 15 and 25 are the only ones belonging to both sets.

(b) $\{2, 3, 4, 5, 6\} \cap \{1, 2, 3, 4\} = \{2, 3, 4\}$

(c) $\{1, 3, 5\} \cap \{2, 4, 6\} = \emptyset$ Disjoint sets

✔ ***Now Try Exercises 59, 65, and 75.***



$A \cup B$

Figure 4

The set of all elements belonging to set A ***or*** to set B (or to both) is called the **union** of the two sets, written $A \cup B$. For example, if $A = \{1, 3, 5\}$ and $B = \{3, 5, 7, 9\}$ then we have the following.

$$A \cup B = \{1, 3, 5\} \cup \{3, 5, 7, 9\} = \{1, 3, 5, 7, 9\}$$

The Venn diagram in **Figure 4** shows two sets A and B. Their union, $A \cup B$, is in color.

EXAMPLE 6 **Finding the Union of Two Sets**

Find each of the following.

(a) $\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\}$

(b) $\{1, 3, 5, 7\} \cup \{2, 4, 6\}$

(c) $\{1, 3, 5, 7, \dots\} \cup \{2, 4, 6, \dots\}$

SOLUTION

(a) Begin by listing the elements of the first set, $\{1, 2, 5, 9, 14\}$. Then include any elements from the second set that are not already listed.

$$\{1, 2, 5, 9, 14\} \cup \{1, 3, 4, 8\} = \{1, 2, 3, 4, 5, 8, 9, 14\}$$

(b) $\{1, 3, 5, 7\} \cup \{2, 4, 6\} = \{1, 2, 3, 4, 5, 6, 7\}$

(c) $\{1, 3, 5, 7, \dots\} \cup \{2, 4, 6, \dots\} = N$ Natural numbers

✔ ***Now Try Exercises 73 and 77.***

The **set operations** are summarized below.

Set Operations

Let A and B be sets, with universal set U.

The **complement** of set A is the set A' of all elements in the universal set that do *not* belong to set A.

$$A' = \{x \mid x \in U, \quad x \notin A\}$$

The **intersection** of sets A and B, written $A \cap B$, is made up of all the elements belonging to both set A *and* set B.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

The **union** of sets A and B, written $A \cup B$, is made up of all the elements belonging to set A *or* to set B.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

R.1 Exercises

Identify each set as finite *or* infinite. *Then determine whether* 10 *is an element of the set.* ***See Example 1.***

1. $\{4, 5, 6, \ldots, 15\}$

2. $\{1, 2, 3, 4, 5, \ldots, 75\}$

3. $\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\}$

4. $\{4, 5, 6, \ldots\}$

5. $\{x \mid x$ is a natural number greater than $11\}$

6. $\{x \mid x$ is a natural number greater than or equal to $10\}$

7. $\{x \mid x$ is a fraction between 1 and $2\}$

8. $\{x \mid x$ is an even natural number$\}$

Use set notation, and list all the elements of each set. ***See Example 2.***

9. $\{12, 13, 14, \ldots, 20\}$

10. $\{8, 9, 10, \ldots, 17\}$

11. $\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{32}\}$

12. $\{3, 9, 27, \ldots, 729\}$

13. $\{17, 22, 27, \ldots, 47\}$

14. $\{74, 68, 62, \ldots, 38\}$

15. $\{x \mid x$ is a natural number greater than 7 and less than $15\}$

16. $\{x \mid x$ is a natural number not greater than $4\}$

Insert $\in$ *or* $\notin$ *in each blank to make the resulting statement true.* ***See Examples 1 and 2.***

17. 6 _____ $\{3, 4, 5, 6\}$

18. 9 _____ $\{3, 2, 5, 9, 8\}$

19. -4 _____ $\{4, 6, 8, 10\}$

20. -12 _____ $\{3, 5, 12, 14\}$

21. 0 _____ $\{2, 0, 3, 4\}$

22. 0 _____ $\{0, 5, 6, 7, 8, 10\}$

23. $\{3\}$ _____ $\{2, 3, 4, 5\}$

24. $\{5\}$ _____ $\{3, 4, 5, 6, 7\}$

25. $\{0\}$ _____ $\{0, 1, 2, 5\}$

26. $\{2\}$ _____ $\{2, 4, 6, 8\}$

27. 0 _____ $\emptyset$

28. $\emptyset$ _____ $\emptyset$

Determine whether each statement is true *or* false. ***See Examples 1–3.***

29. $3 \in \{2, 5, 6, 8\}$

30. $6 \in \{-2, 5, 8, 9\}$

31. $1 \in \{3, 4, 5, 11, 1\}$

32. $12 \in \{18, 17, 15, 13, 12\}$

33. $9 \notin \{2, 1, 5, 8\}$

34. $3 \notin \{7, 6, 5, 4\}$

35. $\{2, 5, 8, 9\} = \{2, 5, 9, 8\}$

36. $\{3, 0, 9, 6, 2\} = \{2, 9, 0, 3, 6\}$

37. $\{5, 8, 9\} = \{5, 8, 9, 0\}$

38. $\{3, 7, 12, 14\} = \{3, 7, 12, 14, 0\}$

39. $\{x \mid x \text{ is a natural number less than } 3\} = \{1, 2\}$

40. $\{x \mid x \text{ is a natural number greater than } 10\} = \{11, 12, 13, \dots\}$

Let $A = \{2, 4, 6, 8, 10, 12\}$, $B = \{2, 4, 8, 10\}$, $C = \{4, 10, 12\}$, $D = \{2, 10\}$, *and* $U = \{2, 4, 6, 8, 10, 12, 14\}$.

Determine whether each statement is true *or* false. ***See Example 3.***

41. $A \subseteq U$

42. $C \subseteq U$

43. $D \subseteq B$

44. $D \subseteq A$

45. $A \subseteq B$

46. $B \subseteq C$

47. $\emptyset \subseteq A$

48. $\emptyset \subseteq \emptyset$

49. $\{4, 8, 10\} \subseteq B$

50. $\{0, 2\} \subseteq D$

51. $B \subseteq D$

52. $A \subseteq C$

Insert $\subseteq$ *or* $\nsubseteq$ *in each blank to make the resulting statement true.* ***See Example 3.***

53. $\{2, 4, 6\}$ ______ $\{3, 2, 5, 4, 6\}$

54. $\{1, 5\}$ ______ $\{0, -1, 2, 3, 1, 5\}$

55. $\{0, 1, 2\}$ ______ $\{1, 2, 3, 4, 5\}$

56. $\{5, 6, 7, 8\}$ ______ $\{1, 2, 3, 4, 5, 6, 7\}$

57. $\emptyset$ ______ $\{1, 4, 6, 8\}$

58. $\emptyset$ ______ $\emptyset$

Determine whether each statement is true *or* false. ***See Examples 4–6.***

59. $\{5, 7, 9, 19\} \cap \{7, 9, 11, 15\} = \{7, 9\}$

60. $\{8, 11, 15\} \cap \{8, 11, 19, 20\} = \{8, 11\}$

61. $\{2, 1, 7\} \cup \{1, 5, 9\} = \{1\}$

62. $\{6, 12, 14, 16\} \cup \{6, 14, 19\} = \{6, 14\}$

63. $\{3, 2, 5, 9\} \cap \{2, 7, 8, 10\} = \{2\}$

64. $\{8, 9, 6\} \cup \{9, 8, 6\} = \{8, 9\}$

65. $\{3, 5, 9, 10\} \cap \emptyset = \{3, 5, 9, 10\}$

66. $\{3, 5, 9, 10\} \cup \emptyset = \{3, 5, 9, 10\}$

67. $\{1, 2, 4\} \cup \{1, 2, 4\} = \{1, 2, 4\}$

68. $\{1, 2, 4\} \cap \{1, 2, 4\} = \emptyset$

69. $\emptyset \cup \emptyset = \emptyset$

70. $\emptyset \cap \emptyset = \emptyset$

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$, $M = \{0, 2, 4, 6, 8\}$, $N = \{1, 3, 5, 7, 9, 11, 13\}$, $Q = \{0, 2, 4, 6, 8, 10, 12\}$, *and* $R = \{0, 1, 2, 3, 4\}$.

Use these sets to find each of the following. Identify any disjoint sets. ***See Examples 4–6.***

71. $M \cap R$

72. $M \cap U$

73. $M \cup N$

74. $M \cup R$

75. $M \cap N$

76. $U \cap N$

77. $N \cup R$

78. $M \cup Q$

79. N'

80. Q'

81. $M' \cap Q$

82. $Q \cap R'$

83. $\emptyset \cap R$

84. $\emptyset \cap Q$

85. $N \cup \emptyset$

86. $R \cup \emptyset$

87. $(M \cap N) \cup R$

88. $(N \cup R) \cap M$

89. $(Q \cap M) \cup R$

90. $(R \cup N) \cap M'$

91. $(M' \cup Q) \cap R$

92. $Q \cap (M \cup N)$

93. $Q' \cap (N' \cap U)$

94. $(U \cap \emptyset') \cup R$

R.2 Real Numbers and Their Properties

- Sets of Numbers and the Number Line
- Exponents
- Order of Operations
- Properties of Real Numbers
- Order on the Number Line
- Absolute Value

Sets of Numbers and the Number Line

As mentioned in the previous section, the set of **natural numbers** is written in set notation as follows.

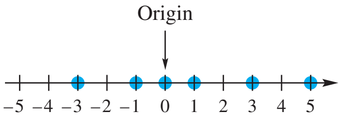
$$\{1, 2, 3, 4, \ldots\} \quad \text{Natural numbers (Section R.1)}$$

Including 0 with the set of natural numbers gives the set of **whole numbers.**

$$\{0, 1, 2, 3, 4, \ldots\} \quad \text{Whole numbers}$$

Including the negatives of the natural numbers with the set of whole numbers gives the set of **integers.**

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \quad \text{Integers}$$



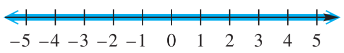
Graph of the Set {−3, −1, 0, 1, 3, 5}

Figure 5

Integers can be **graphed** on a **number line.** See **Figure 5**. Every number corresponds to one and only one point on the number line, and each point corresponds to one and only one number. The number associated with a given point is called the **coordinate** of the point. This correspondence forms a **coordinate system.**

The result of dividing two integers (with a nonzero divisor) is called a *rational number,* or *fraction.* A **rational number** is an element of the set defined as follows.

$$\left\{ \frac{p}{q} \,\middle|\, p \text{ and } q \text{ are integers and } q \neq 0 \right\} \quad \text{Rational numbers}$$



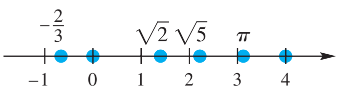
Graph of the Set of Real Numbers

Figure 6

The set of rational numbers includes the natural numbers, the whole numbers, and the integers. For example, the integer -3 is a rational number because it can be written as $\frac{-3}{1}$. Numbers that can be written as repeating or terminating decimals are also rational numbers. For example, $0.\overline{6} = 0.66666\ldots$ represents a rational number that can be expressed as the fraction $\frac{2}{3}$.

The set of all numbers that correspond to points on a number line is the **real numbers,** shown in **Figure 6**. Real numbers can be represented by decimals. Since every fraction has a decimal form—for example, $\frac{1}{4} = 0.25$—real numbers include rational numbers.

Some real numbers cannot be represented by quotients of integers. These numbers are **irrational numbers.** The set of irrational numbers includes $\sqrt{3}$ and $\sqrt{5}$. Another irrational number is π, which is *approximately* equal to 3.14159. The numbers in the set $\left\{-\frac{2}{3}, 0, \sqrt{2}, \sqrt{5}, \pi, 4\right\}$ can be located on a number line, as shown in **Figure 7**.



$\sqrt{2}$, $\sqrt{5}$, and π are irrational. Since $\sqrt{2}$ is approximately equal to 1.41, it is located between 1 and 2, slightly closer to 1.

Figure 7

The sets of numbers discussed so far are summarized as follows.

Sets of Numbers

Set	Description
Natural numbers	$\{1, 2, 3, 4, \ldots\}$
Whole numbers	$\{0, 1, 2, 3, 4, \ldots\}$
Integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational numbers	$\left\{\frac{p}{q} \mid p \text{ and } q \text{ are integers and } q \neq 0\right\}$
Irrational numbers	$\{x \mid x \text{ is real but not rational}\}$
Real numbers	$\{x \mid x \text{ corresponds to a point on a number line}\}$

EXAMPLE 1 Identifying Sets of Numbers

Let $A = \left\{-8, -6, -\frac{12}{4}, -\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}, 1, \sqrt{2}, \sqrt{5}, 6\right\}$. List the elements from A that belong to each set.

(a) Natural numbers **(b)** Whole numbers **(c)** Integers

(d) Rational numbers **(e)** Irrational numbers **(f)** Real numbers

SOLUTION

(a) Natural numbers: 1 and 6 **(b)** Whole numbers: 0, 1, and 6

(c) Integers: $-8, -6, -\frac{12}{4}$ (or -3), 0, 1, and 6

(d) Rational numbers: $-8, -6, -\frac{12}{4}$ (or -3), $-\frac{3}{4}, 0, \frac{3}{8}, \frac{1}{2}$, 1, and 6

(e) Irrational numbers: $\sqrt{2}$ and $\sqrt{5}$

(f) All elements of A are real numbers.

✔ *Now Try Exercises 1, 11, and 13.*

Figure 8 shows the relationships among the subsets of the real numbers.

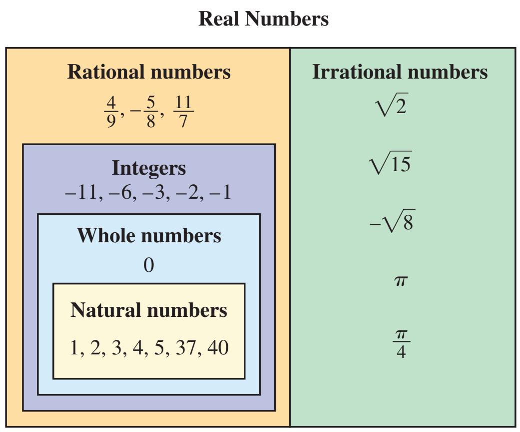


Figure 8

Exponents The product $2 \cdot 2 \cdot 2$ can be written as 2^3, where the 3 shows that three factors of 2 appear in the product.

Exponential Notation

If n is any positive integer and a is any real number, then the nth power of a is written using exponential notation as follows.

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a}$$

That is, a^n means the product of n factors of a. The integer n is the **exponent,** a is the **base,** and a^n is a **power** or an **exponential expression** (or simply an **exponential**). Read a^n as **"*a* to the *n*th power,"** or just **"*a* to the *n*th."**

EXAMPLE 2 Evaluating Exponential Expressions

Evaluate each exponential expression, and identify the base and the exponent.

(a) 4^3 **(b)** $(-6)^2$ **(c)** -6^2 **(d)** $4 \cdot 3^2$ **(e)** $(4 \cdot 3)^2$

SOLUTION

(a) $4^3 = \underbrace{4 \cdot 4 \cdot 4}_{3 \text{ factors of } 4} = 64$ The base is 4 and the exponent is 3.

(b) $(-6)^2 = (-6)(-6) = 36$ The base is -6 and the exponent is 2.

(c) $-6^2 = -(6 \cdot 6) = -36$ Notice that parts (b) and (c) are different.
The base is 6 and the exponent is 2.

(d) $4 \cdot 3^2 = 4 \cdot 3 \cdot 3 = 36$ The base is 3 and the exponent is 2.
$3^2 = 3 \cdot 3$, NOT $3 \cdot 2$

(e) $(4 \cdot 3)^2 = 12^2 = 144$ $(4 \cdot 3)^2 \neq 4 \cdot 3^2$
The base is $4 \cdot 3$, or 12, and the exponent is 2.

✔ *Now Try Exercises 17, 19, 21, and 23.*

Order of Operations When a problem involves more than one operation symbol, we use the following order of operations.

Order of Operations

If grouping symbols such as parentheses, square brackets, absolute value bars, or fraction bars are present, begin as follows.

Step 1 Work separately above and below each **fraction bar.**

Step 2 Use the rules below within each set of **parentheses** or **square brackets.** Start with the innermost set and work outward.

If no grouping symbols are present, follow these steps.

Step 1 Simplify all **powers** and **roots.** ***Work from left to right.***

Step 2 Do any **multiplications** or **divisions** in order. ***Work from left to right.***

Step 3 Do any **negations, additions,** or **subtractions** in order. ***Work from left to right.***

EXAMPLE 3 Using Order of Operations

Evaluate each expression.

(a) $6 \div 3 + 2^3 \cdot 5$

(b) $(8 + 6) \div 7 \cdot 3 - 6$

(c) $\dfrac{4 + 3^2}{6 - 5 \cdot 3}$

(d) $\dfrac{-(-3)^3 + (-5)}{2(-8) - 5(3)}$

SOLUTION

(a)
$$\begin{aligned} 6 \div 3 + 2^3 \cdot 5 &= 6 \div 3 + 8 \cdot 5 && \text{Evaluate the exponential.} \\ &= 2 + 8 \cdot 5 && \text{Divide.} \\ &= 2 + 40 && \text{Multiply.} \\ &= 42 && \text{Add.} \end{aligned}$$

Multiply or divide *in order from left to right.*

(b) $(8+6) \div 7 \cdot 3 - 6 = 14 \div 7 \cdot 3 - 6$ Work inside parentheses.

Be careful to divide *before* multiplying here.

$= 2 \cdot 3 - 6$ Divide.

$= 6 - 6$ Multiply.

$= 0$ Subtract.

(c) $\frac{4+3^2}{6-5\cdot 3} = \frac{4+9}{6-15}$ Evaluate the exponential and multiply.

$= \frac{13}{-9}$, or $-\frac{13}{9}$ Add and subtract; $\frac{a}{-b} = -\frac{a}{b}$.

(d) $\frac{-(-3)^3 + (-5)}{2(-8) - 5(3)} = \frac{-(-27) + (-5)}{2(-8) - 5(3)}$ Evaluate the exponential.

$= \frac{27 + (-5)}{-16 - 15}$ Multiply.

$= \frac{22}{-31}$, or $-\frac{22}{31}$ Add and subtract; $\frac{a}{-b} = -\frac{a}{b}$.

✔ ***Now Try Exercises 25, 27, and 33.***

EXAMPLE 4 Using Order of Operations

Evaluate each expression for $x = -2$, $y = 5$, and $z = -3$.

(a) $-4x^2 - 7y + 4z$ **(b)** $\frac{2(x-5)^2 + 4y}{z+4}$ **(c)** $\frac{\frac{x}{2} - \frac{y}{5}}{\frac{3z}{9} + \frac{8y}{5}}$

SOLUTION

Use parentheses around substituted values to avoid errors.

(a) $-4x^2 - 7y + 4z = -4(-2)^2 - 7(5) + 4(-3)$ Substitute: $x = -2$, $y = 5$, and $z = -3$.

$= -4(4) - 7(5) + 4(-3)$ Evaluate the exponential.

$= -16 - 35 - 12$ Multiply.

$= -63$ Subtract.

(b) $\frac{2(x-5)^2 + 4y}{z+4} = \frac{2(-2-5)^2 + 4(5)}{-3+4}$ Substitute: $x = -2$, $y = 5$, and $z = -3$.

$= \frac{2(-7)^2 + 20}{1}$ Work inside parentheses. Then multiply and add.

$= 2(49) + 20$ Evaluate the exponential.

$= 98 + 20$ Multiply.

$= 118$ Add.

(c) $\frac{\frac{x}{2} - \frac{y}{5}}{\frac{3z}{9} + \frac{8y}{5}} = \frac{\frac{-2}{2} - \frac{5}{5}}{\frac{3(-3)}{9} + \frac{8(5)}{5}}$ Substitute: $x = -2$, $y = 5$, and $z = -3$.

$= \frac{-1-1}{-1+8}$, or $-\frac{2}{7}$ Simplify the fractions.

✔ ***Now Try Exercises 35, 43, and 45.***

Properties of Real Numbers The following basic properties can be generalized to apply to expressions with variables.

Properties of Real Numbers

Let a, b, and c represent real numbers.

Property	Description
Closure Properties $a + b$ is a real number. ab is a real number.	The sum or product of two real numbers is a real number.
Commutative Properties $a + b = b + a$ $ab = ba$	The sum or product of two real numbers is the same regardless of their order.
Associative Properties $(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$	The sum or product of three real numbers is the same no matter which two are added or multiplied first.
Identity Properties There exists a unique real number 0 such that $a + 0 = a$ and $0 + a = a.$ There exists a unique real number 1 such that $a \cdot 1 = a$ and $1 \cdot a = a.$	The sum of a real number and 0 is that real number, and the product of a real number and 1 is that real number.
Inverse Properties There exists a unique real number $-a$ such that $a + (-a) = 0$ and $-a + a = 0.$ If $a \neq 0$, there exists a unique real number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1.$	The sum of any real number and its negative is 0, and the product of any nonzero real number and its reciprocal is 1.
Distributive Properties $a(b + c) = ab + ac$ $a(b - c) = ab - ac$	The product of a real number and the sum (or difference) of two real numbers equals the sum (or difference) of the products of the first number and each of the other numbers.

The **multiplication property of zero** says that $\mathbf{0 \cdot a = a \cdot 0 = 0}$ for all real numbers a.

CAUTION With the commutative properties, the ***order*** changes, but with the associative properties, the ***grouping*** changes.

Commutative Properties	Associative Properties
$(x + 4) + 9 = (4 + x) + 9$	$(x + 4) + 9 = x + (4 + 9)$
$7 \cdot (5 \cdot 2) = (5 \cdot 2) \cdot 7$	$7 \cdot (5 \cdot 2) = (7 \cdot 5) \cdot 2$

EXAMPLE 5 Simplifying Expressions

Use the commutative and associative properties to simplify each expression.

(a) $6 + (9 + x)$ **(b)** $\frac{5}{8}(16y)$ **(c)** $-10p\left(\frac{6}{5}\right)$

SOLUTION

(a) $6 + (9 + x) = (6 + 9) + x$ Associative property

$= 15 + x$ Add.

(b) $\frac{5}{8}(16y) = \left(\frac{5}{8} \cdot 16\right)y$ Associative property

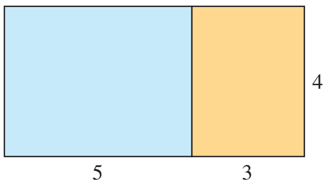
$= 10y$ Multiply.

(c) $-10p\left(\frac{6}{5}\right) = \frac{6}{5}(-10p)$ Commutative property

$= \left[\frac{6}{5}(-10)\right]p$ Associative property

$= -12p$ Multiply.

✔ ***Now Try Exercises 63 and 65.***



Geometric Model of the Distributive Property

Figure 9

Figure 9 helps to explain the distributive property. The area of the entire region shown can be found in two ways, as follows.

$$4(5 + 3) = 4(8) = 32$$

or
$$4(5) + 4(3) = 20 + 12 = 32$$

The result is the same. This means that

$$4(5 + 3) = 4(5) + 4(3).$$

EXAMPLE 6 Using the Distributive Property

Rewrite each expression using the distributive property and simplify, if possible.

(a) $3(x + y)$ **(b)** $-(m - 4n)$ **(c)** $\frac{1}{3}\left(\frac{4}{5}m - \frac{3}{2}n - 27\right)$ **(d)** $7p + 21$

SOLUTION

(a) $3(x + y) = 3x + 3y$

(b) $-(m - 4n) = -1(m - 4n)$

$= -1(m) + (-1)(-4n)$

$= -m + 4n$

Be careful with the negative signs.

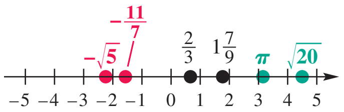
(c) $\frac{1}{3}\left(\frac{4}{5}m - \frac{3}{2}n - 27\right) = \frac{1}{3}\left(\frac{4}{5}m\right) + \frac{1}{3}\left(-\frac{3}{2}n\right) + \frac{1}{3}(-27)$

$= \frac{4}{15}m - \frac{1}{2}n - 9$

(d) $7p + 21 = 7p + 7 \cdot 3$

$= 7(p + 3)$ Distributive property in reverse

✔ ***Now Try Exercises 67, 69, and 71.***



$-\sqrt{5}$ is to the left of $-\frac{11}{7}$ on the number line, so $-\sqrt{5} < -\frac{11}{7}$, and $\sqrt{20}$ is to the right of π, indicating that $\sqrt{20} > \pi$.

Figure 10

Order on the Number Line If the real number a is to the left of the real number b on a number line, then

$$\textbf{\textit{a} is less than \textit{b},} \quad \text{written} \quad a < b.$$

If a is to the right of b, then

$$\textbf{\textit{a} is greater than \textit{b},} \quad \text{written} \quad a > b.$$

The inequality symbol must point toward the lesser number.

Figure 10 illustrates this with several pairs of numbers. Statements involving these symbols, as well as the symbols less than or equal to, $\leq$, and greater than or equal to, $\geq$, are called **inequalities.** The inequality $a < b < c$ says that b is ***between*** a and c since $a < b$ and $b < c$.

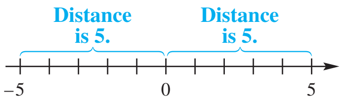


Figure 11

Absolute Value The distance on the number line from a number to 0 is called the **absolute value** of that number. The absolute value of the number a is written $|a|$. For example, the distance on the number line from 5 to 0 is 5, as is the distance from -5 to 0. See **Figure 11.** Therefore, both of the following are true.

$$|5| = 5 \quad \text{and} \quad |-5| = 5$$

NOTE ***Since distance cannot be negative, the absolute value of a number is always positive or 0.***

The algebraic definition of absolute value follows.

Absolute Value

Let a represent a real number.

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

That is, the absolute value of a positive number or 0 equals that number, while the absolute value of a negative number equals its negative (or opposite).

EXAMPLE 7 Evaluating Absolute Values

Evaluate each expression.

(a) $\left|-\frac{5}{8}\right|$ **(b)** $-|8|$ **(c)** $-|-2|$ **(d)** $|2x|$, for $x = \pi$

SOLUTION

(a) $\left|-\frac{5}{8}\right| = \frac{5}{8}$

(b) $-|8| = -(8) = -8$

(c) $-|-2| = -(2) = -2$

(d) $|2\pi| = 2\pi$

✔ ***Now Try Exercises 83 and 87.***

Absolute value is useful in applications where only the *size* (or magnitude), not the *sign,* of the difference between two numbers is important.